

Ratchet effects for paramagnetic beads above striped ferrite-garnet films

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We calculate the motion of a small paramagnetic bead which is manipulated by the stripe domain pattern of a ferrite-garnet film. A model for the bead's motion in a liquid above the film is developed and used to look for ratchet solutions, where the bead acquires net coherent motion in one direction when the external field is modulated periodically. We consider three cases. First, the ratchet, where the beads all go in the same direction. Second, the height dependent ratchet, where beads at different heights go in opposite direction. This case can be used to separate beads of different sizes, as considered in J. Phys. Chem. B 112, 3833 (2008). Third, we describe how the separation threshold can be tuned by changing the amplitude of the applied field. Finally, we describe a pseudo ratchet, where the external modulation is not periodic and the ratchet changes direction periodically.

I. INTRODUCTION

Functionalized micrometer-sized beads have for a long time been used for medical and biological applications where a typical application is to attach biologically active molecules to the beads and use the beads as carriers. However, the setup for such applications is usually limited to bulk manipulation where the beads are contained in an aqueous solution, and such setups do not offer precise positioning of individual beads. Recently this situation has improved and several devices demonstrating controlled manipulation of a small number of beads have been realized, for example by using a thin zig-zag electric wire¹ or by a hybrid device magnetic bead separator device using current wires and magnetic fields combined with microfluidic channels.²

One promising way to realize devices that manipulate individual beads is to use domain patterns in ferrite garnet films. (For a review, see. Ref. 3) Because the paramagnetic beads get strongly pinned to the domain walls the influence of thermal noise and other perturbations is small. Also, the beads can easily be moved from the domain walls by an external field interacting both with the beads and the domain pattern. An additional level of control can be achieved by monitoring the domain patterns by exploiting the larger Faraday rotation of ferrite garnet films. All these features were demonstrated in Ref. 4 in which the stripe pattern of a ferrite garnet film in an harmonic applied field created a magnetic ratchet effect, where beads jumped from domain wall to domain wall in just one direction. In the same system, with a more complex applied field, Ref. 5 demonstrated an even more surprising effect: beads of different sizes go in opposite directions. The motion in this double ratchet, as as in the original ratchet, was synchronized so beads jump only one wavelength of the stripe pattern per period of the external field.

In this work we model the the system of Refs. 4 and 5 and discuss the cause of the effects and tuning of the devices.

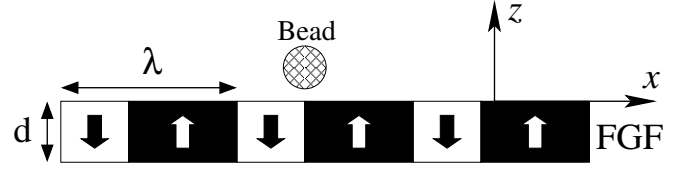


FIG. 1. A side-view of the stripe domain pattern of a ferrite garnet film. The film has thickness d and pattern wavelength is λ . A paramagnetic bead with radius a is pinned to the domain wall.

II. MODEL

Consider a paramagnetic bead dispensed in a liquid above the surface of the film. The bead motion is mainly determined by the hydrodynamic force F_x^h and magnetic force F_x^m . Then the equation of motion is

$$F_x^h + F_x^m = 0. \quad (1)$$

This overdamped motion is a reasonable approximation for slow dynamics and it is also reasonable to neglect other forces, provided the beads avoid direct contact with the surface. The hydrodynamic drag of slowly moving beads dispensed in a liquid is quantified by Stoke's law

$$F_x^h = 6\pi f a \eta \dot{x}, \quad (2)$$

where a is bead radius, η is the dynamic viscosity of water and f is a correction factor due to the presence of the film, $f = f(z) \geq 1$. In experiments the film is typically electrostatically charged to prevent sticking, so that the beads levitate a distance a few nanometer above the film, yielding $f < 3$.⁶

The magnetic force on the paramagnetic bead with volume V and magnetic susceptibility χ is

$$F_x^m = -\frac{\partial U}{\partial x}. \quad (3)$$

with potential

$$U = -\frac{1}{2}\mu V \chi H^2 \quad (4)$$

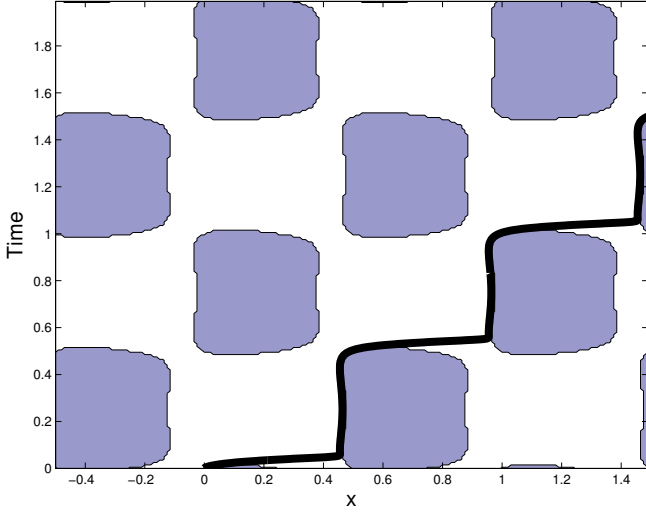


FIG. 2. The ratchet effect. The horizontal axis is the position x and the vertical axis is time t . The particle path is plotted as a solid line above the landscape formed by the sign of F_x^m . Blue means $F_x^m < 0$ and white is $F_x^m > 0$.

where $\mu \approx \mu_0 = 1.26 \times 10^{-6}$ Tm/A is the permeability of water,

Consider a magnetic film with stripe domains. Above the surface of the film, the total magnetic field is

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}^a(t) + \mathbf{H}^f(\mathbf{r}, t) \quad (5)$$

where \mathbf{H}^a is applied field and \mathbf{H}^f is the inhomogeneous field from the stripe pattern of the film. The applied field is periodic, with

$$\begin{aligned} H_z^a &= H_0 \sin(2\pi\nu t) \\ H_x^a &= sH_0 \sin(g2\pi\nu t) \end{aligned} \quad (6)$$

where g is an integer and s is an arbitrary number.

For a film with stripe domains and magnetization M_s , the magnetic field above the film can be expressed as

$$\begin{aligned} H_x^f &= \frac{M_s}{\pi} \operatorname{Re} \psi \\ H_z^f &= \frac{M_s}{\pi} \operatorname{Im} \psi \end{aligned} \quad (7)$$

where the complex auxiliary function ψ is

$$\begin{aligned} \psi &= \log \left(\frac{1 - e^{-2\pi(z+i(x+\Delta))/\lambda}}{1 + e^{-2\pi(z-i(x-\Delta))/\lambda}} \right) \\ &+ \log \left(\frac{1 + e^{-2\pi(z+d+i(x+\Delta))/\lambda}}{1 - e^{-2\pi(z+d+i(x-\Delta))/\lambda}} \right). \end{aligned} \quad (8)$$

where d is film thickness, λ is the wavelength of the stripe pattern and Δ is the domain wall displacement. The second term in Eq. (8) comes from the bottom surface of the film and it can be neglected when beads are close to the film, i.e., when $z \ll d$. The domain wall displacement depends on the applied field, and for small and

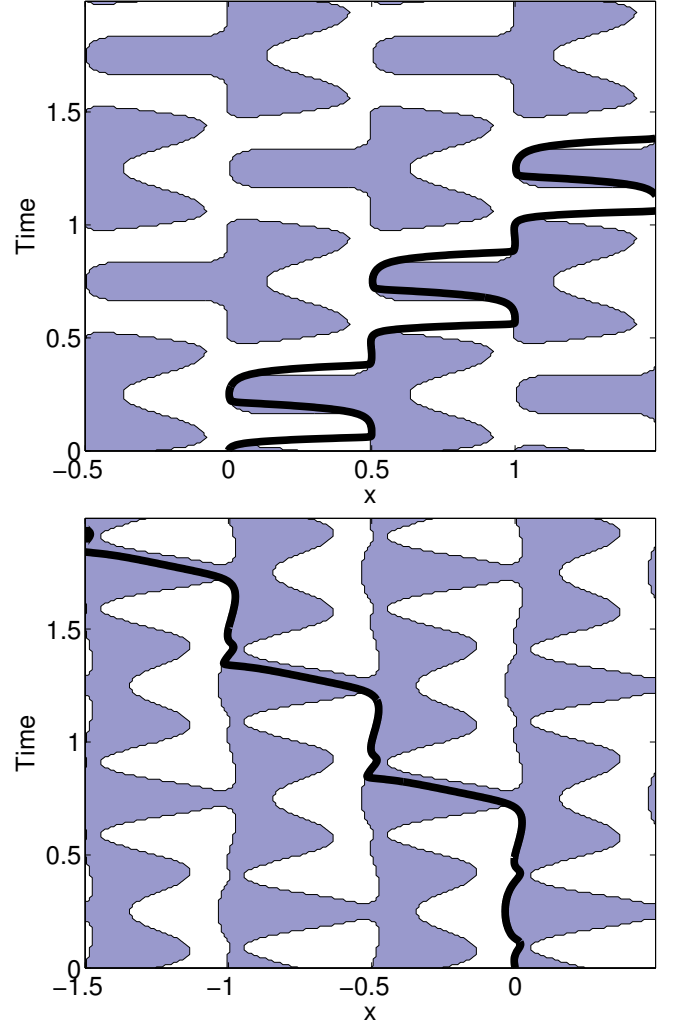


FIG. 3. Separation effect, in which beads at different heights move in opposite directions. The bead's trajectory is plotted above the sign of the interaction force. Top: $z = 0.1\lambda$, bottom: $z = 0.05\lambda$. All other parameters are equal: $H_0 = 0.18M_s$ and $\Delta_0 = 0.05$.

medium amplitudes we assume it to be proportional to H_z^a , i.e.

$$\Delta(t) = \Delta_0 \frac{H_z^a(t)}{H_0}. \quad (9)$$

III. RESULTS

When the external field is modulated there will be response in the width of the strip domains and the paramagnetic bead will move according to Eq. (1). We will now consider several solutions: the ratchet, the pseudo ratchet and the size dependent ratchet.

Ratchet - An example of coherent dynamics is the ratchet. In the ratchet the paramagnetic beads gain a net coherent motion in one direction, despite a periodic

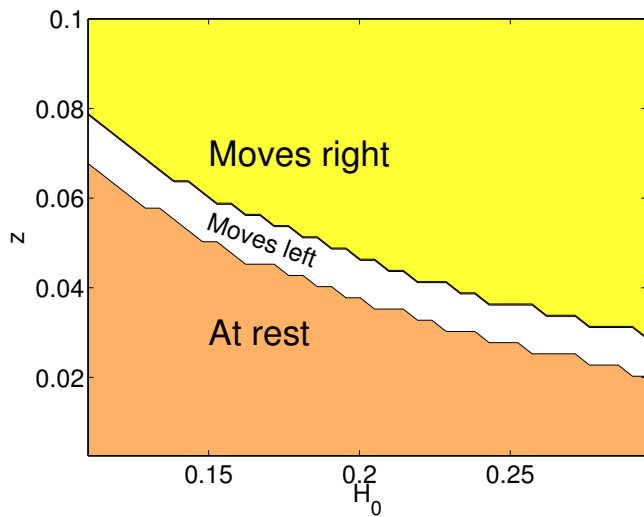


FIG. 4. Tuning of the separation height by changing the strength of the applied field H_0 from Eq. (6). The part with inverted ratchet direction is seen as a white line.

modulation of the external field. The bead's motion is plotted in Fig. 2. The external field is in this setup constantly at angle 45° corresponding to $s = 1$ and $g = 1$ in Eq. (6). The motion of the will nonetheless acquire a coherent motion to the right, as seen in Fig. 2.

Separation - Complex modulation of \mathbf{H}_a opens the possibility even more strange ratchet solutions of Eq. (1), for example ratchets that go in opposite directions at different heights. This effect was exploited in Ref. 5 to yield separation of beads of different sizes. The height dependent ratchet appears for $g = 3$ in Eq. (6) and the results trajectories for beads at heights $z = 0.1\lambda$ and $z = 0.05\lambda$ are plotted in Fig. 3. There is a clear difference: the large bead jumps three times during a half-period and acquires a net motion to the right while the small bead jumps only once and acquires a net motion to the left, all in perfect agreement with Ref. 5. The two trajectories are entirely as expected from the sign of F_x which are plotted in the background. However, we notice that the beads do not follow the edge of the signs exactly, because of the finite viscosity. Only in the limit $\eta \rightarrow 0$, the beads would follow the edges exactly.

Tuning of separation threshold - The height dependent ratchet will only be useful as a separation device if the separation threshold z_{th} is possible to tune. The only realistic parameter to change is \mathbf{H}_a , but since the film reacts to H_z^a the most robust approach is to keep H_z^a fixed and just change H_x^a . The effect of changing s is seen in Fig. 5 which is plotted in the style of phase diagram, distinguishing three outcomes: motion to the left, to the right and at rest.

Pseudo ratchet - The ratchet effect is a consequence of phase coherence between the H_x^a and H_z^a and the ratchet fails when there is a drift in H_x^a , i.e., when g from Eq. (6) is not an integer. We will no consider the pseudo ratchet

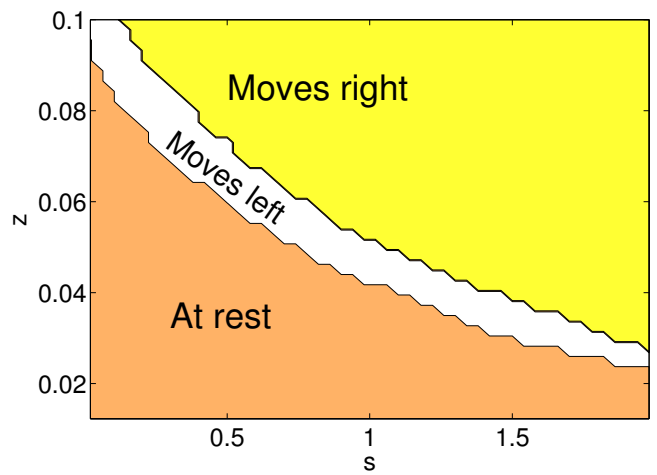


FIG. 5. Tuning of the separation height by the amplitude asymmetry of the applied field s from Eq. (6). $H_0 = 0.18M_s$. The part with inverted ratchet direction is seen as a white line.

solutions which appear when

$$g = 1 + 1/N \quad (10)$$

where N is an integer. Since $\sin(\omega(1 + 1/N)t) = \sin(\omega t) \cos(\omega/N t) + \cos(\omega t) \sin(\omega/N t)$ we can expect to see some effect of some convolution with the longer period N/ω , but what is surprising is that the coherent dynamics is intact. We then get the pseudo ratchet solutions of Fig. 6 for $N = 2, 3, 4$ and 5 in which the ratchet changes direction after N periods.

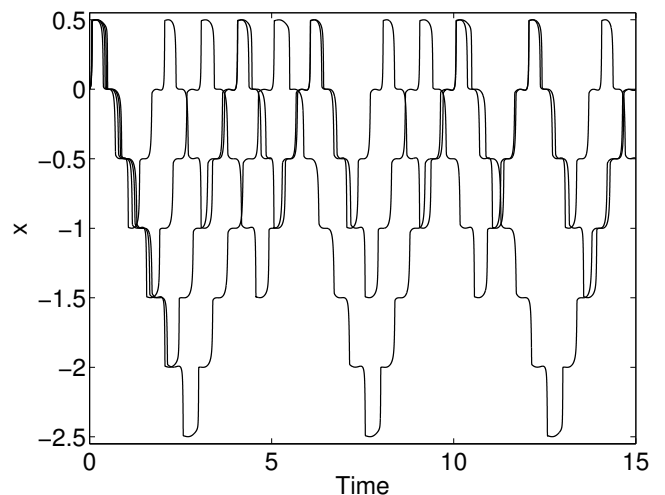


FIG. 6. Pseudo ratchet, in which the ratchet direction changes after a $N = 2, 3, 4$ and 5 periods.

IV. SUMMARY

We have done theoretical modeling of the dynamics of small paramagnetic beads manipulated by the stripe domain pattern of a ferrite garnet film. The simple model, only taking into account the paramagnetic interaction and linear hydrodynamics, is sufficient to explain several non-trivial dynamical phenomenon occurring in the

system. In particular the model is capable of explaining the rectified motion occurring in a harmonic external field and also the more complicated separation effect, in which a driving with different frequencies in x and z direction cause beads of different heights to go in different directions. We find that this effect can be explained as a consequence of an asymmetry in the magnetic interaction potential.

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